Dominating Sets and Domination Polynomials of Square of Ladder

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Abstract— Let G = (V, E) be a simple graph. A set S $\subseteq$ V is a dominating set of G, if every vertex in V-S is adjacent to atleast one vertex in S. Let L_n^2 be the square of Ladder graph and let $D(L_n^2, i)$ denote the family of all dominating sets of L_n^2 with cardinality i. Let $d(L_n^2, i) = \left|D(L_n^2, i)\right|$. In this paper, we obtain a recursive formula for $d(L_n^2, i)$ and study the dominating sets of L_n^2 .Using this formula, we construct the polynomial, $D(L_n^2, i) = \sum_{i=\left\lfloor \frac{n}{4} \right\rfloor + 1}^{n} d(L_n^2, i)x^i$ which we call domination polynomial of L_n^2 and some properties of this polynomial are studied.

Index Terms— domination set, domination number, domination polynomials.

1 Introduction

LET G = (V,E) be a simple graph of order |V| = n. For any vertex v$\in$V, the open neighbourhood of v is the set N (v) = {u$\in$V/uv$\in$E} and the closed neighbourhood of v is the set N[v] = N (v) $\cup${v}. For a set S $\subseteq$ V, the open neighbourhood of S is N(S) = $\cup_{v\in S}$ N (v) and the closed neighbourhood of S is N[S] = N(S) $\cup$ S. A set S $\subseteq$ V is a dominating set of G, if N[S] = V, or equivalently, every vertex in V-S is adjacent to atleast one vertex in S. The domination number of a graph G is defined as the minimum size of a dominating set of vertices in G and it is denoted as $\gamma(G)$.

Definition: 1.1

The Second power of a graph with the same set of vertices as G and an edge between two vertices if and only if there is a path of length atmost 2 between them.

Definition: 1.2

Consider two paths $[a_1, a_2, ..., a_k]$ and $[b_1, b_2, ..., b_k]$,join each pair of vertices $a_i, b_i, i = 1, 2, ..., k$ with a new path. The resulting graph is called a ladder, and the path between are called its rungs.

As usual we use $\lfloor x \rfloor$ for the largest integer less than or equal to x and $\lceil x \rceil$ for the smallest integer greater than or equal to x. Also, we denote the set {1, 2,… n} by [n], throughout this paper.

2 DOMINATING SETS OF SQUARE OF LADDER

For the construction of the dominating sets of the square of ladder, L_n^2, we need to investigate the dominating sets of $L_n^2 - \{2n\}$.In this section we investigate dominating sets of L_n^2. Let $D(L_n^2, i)$ be the family of dominating sets of L_n^2 with cardinality i. We shall find recursive formula for $\left|D(L_n^2, i)\right|$ .We need the following lemmas to obtain the result of this section:

Lemma 2.1:

$$\gamma(P_n^2) = \left\lceil \frac{n}{5} \right\rceil$$

Proof: By theorem in [5]

Lemma 2.2

For every $n \in N$,

i) $\gamma(L_n^2) = \left\lfloor \frac{n}{4} \right\rfloor + 1$

ii) $\gamma(L_n^2 - \{2n\}, i) = \left\lfloor \frac{n}{4} \right\rfloor + 1$

iii) $D(L_n^2, i) = \Phi$ if and only if $i \prec \left\lfloor \frac{n}{4} \right\rfloor + 1$ or $i \succ 2n$

iv) $\gamma(L_n^2 - \{2n\}, i) = \Phi$ if and only if $i \prec \left\lfloor \frac{n}{4} \right\rfloor + 1$ or $i \succ 2n - 1$

Proof: It follows from the definition of domination number and lemma 2.1.

Lemma 2.3

i) If $D(L_n^2 - \{2n\}, i-1) = D(L_{n-1}^2, i-1) = D(L^2_{n-1} - \{2n-2\}, i-1) = \Phi$
$D(L_{n-2}^2, i-1) = \Phi$, $D(L^2_{n-2} - \{2n-4\}, i-1) = D(L_{n-3}^2, i-1) = \Phi$
$D(L_{n-4}^2, i-1) = \Phi$, then $D(L_n^2, i) = \Phi$.

ii) If $D(L_n^2 - \{2n\}, i-1) = D(L_{n-1}^2, i-1) = D(L^2_{n-1} - \{2n-2\}, i-1) = \Phi$
$D(L_{n-2}^2, i-1) = D(L^2_{n-2} - \{2n-4\}, i-1) = D(L_{n-3}^2, i-1) = \Phi$

and $D(L_{n-4}^2, i-1) \neq \Phi$,then $D(L_n^2, i) \neq \Phi$.

Proof

i) Since, $D(L_n^2 - \{2n\}, i-1) = D(L_{n-1}^2, i-1) = \Phi$

$D(L^2_{n-1} - \{2n-2\}, i-1) = D(L_{n-2}^2, i-1) = \Phi$,

$D(L^2_{n-2} - \{2n-4\}, i-1) = \Phi$

$D(L_{n-3}^2, i-1) = D(L_{n-4}^2, i-1) = \Phi$, by lemma 2.2(iii),(iv) ,

$i-1 \prec \left\lfloor \frac{n}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-1, $i-1 \prec \left\lfloor \frac{n-1}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-2,

$i-1 \prec \left\lfloor \frac{n-1}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-3,

$i-1 \prec \left\lfloor \frac{n-2}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-4, $i-1 \prec \left\lfloor \frac{n-2}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-5,

$i-1 \prec \left\lfloor \frac{n-3}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-6

and $i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-8.

Therefore, we have $i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-1

Therefore, $i \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 2$ or i $\succ 2n$

That is $i \prec \left\lfloor \frac{n}{4} \right\rfloor + 1$ or i $\succ 2n$

Therefore, $D(L_n^2, i) = \Phi$.

ii) Since , $D(L_n^2 - \{2n\}, i-1) = D(L_{n-1}^2, i-1) =$

$D(L^2_{n-1} - \{2n-2\}, i-1) = \Phi$

$D(L_{n-2}^2, i-1) = \Phi$, $D(L^2_{n-2} - \{2n-4\}, i-1) =$

$D(L_{n-3}^2, i-1) = \Phi$

and $D(L_{n-4}^2, i-1) \neq \Phi$,by lemma 2.2(iii),(iv),

We have

$i-1 \prec \left\lfloor \frac{n}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-1, $i-1 \prec \left\lfloor \frac{n-1}{4} \right\rfloor + 1$ or

$i-1 \succ 2n$-2, $i-1 \prec \left\lfloor \frac{n-1}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-3,

$i-1 \prec \left\lfloor \frac{n-2}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-4, $i-1 \prec \left\lfloor \frac{n-2}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-5,

$i-1 \prec \left\lfloor \frac{n-3}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-6 and

$\left\lfloor \frac{n-4}{4} \right\rfloor + 1 \leq i-1 \leq 2n$-8.

From the above inequalities, we have $i-1 \prec \left\lfloor \frac{n-3}{4} \right\rfloor + 1$ or

$i-1 \succ 2n$-1 and

$\left\lfloor \frac{n-4}{4} \right\rfloor + 1 \leq i-1 \leq 2n$-8.

As $i-1 \leq 2n-8$, $i-1 \leq 2n-1$.

Therefore, $i-1 \prec \left\lfloor \frac{n-3}{4} \right\rfloor + 1$ and $\left\lfloor \frac{n-4}{4} \right\rfloor + 1 \leq i-1 \leq 2n$-8.

Therefore, $\left\lfloor \frac{n-4}{4} \right\rfloor + 1 \leq i-1 \prec \left\lfloor \frac{n-3}{4} \right\rfloor + 1$.

Therefore $\left\lfloor \frac{n-4}{4} \right\rfloor + 2 \leq i \prec \left\lfloor \frac{n-3}{4} \right\rfloor + 2$.

But $\left\lfloor \frac{n}{4} \right\rfloor + 1 \leq \left\lfloor \frac{n-4}{4} \right\rfloor + 2$

Therefore, $i \geq \left\lfloor \frac{n}{4} \right\rfloor + 1$

Therefore, $D(L_n^2, i) \neq \Phi$.

Theorem 2.4
For every n

i) If $D(L_n^2 - \{2n\}, i-1) \neq \Phi$ and $D(L_{n-1}^2, i-1) = \Phi$

$D(L^2_{n-1} - \{2n-2\}, i-1) = D(L_{n-2}^2, i-1) =$

$D(L^2_{n-2} - \{2n-4\}, i-1) = \Phi$

$D(L_{n-3}^2, i-1) = D(L_{n-4}^2, i-1) = \Phi$ iff i=2n.

ii) If $D(L_n^2 - \{2n\}, i-1) \neq \Phi$, $D(L_{n-1}^2, i-1) \neq \Phi$ and

$D(L^2_{n-1} - \{2n-2\}, i-1) = \Phi$ $D(L_{n-2}^2, i-1) =$

$D(L^2_{n-2} - \{2n-4\}, i-1) = \Phi$

$D(L_{n-3}^2, i-1) = D(L_{n-4}^2, i-1) = \Phi$ iff i=2n-1

iii) If $D(L_n^2 - \{2n\}, i-1) \neq \Phi$, $D(L_{n-1}^2, i-1) \neq \Phi$,

$D(L^2_{n-1} - \{2n-2\}, i-1) \neq \Phi$ and $D(L_{n-2}^2, i-1) = \Phi$,

$D(L^2_{n-2} - \{2n-4\}, i-1) = \Phi$ $D(L_{n-3}^2, i-1) =$

$D(L_{n-4}^2, i-1) = \Phi$ iff i=2n-2

iv) If $D(L_n^2 - \{2n\}, i-1) \neq \Phi$, $D(L_{n-1}^2, i-1) \neq \Phi$,

$D(L^2_{n-1} - \{2n-2\}, i-1) \neq \Phi$, $D(L_{n-2}^2, i-1) \neq \Phi$ and

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$D(L^2_{n-2}-\{2n-4\},i-1)=D(L^2_{n-3},i-1)=D(L^2_{n-4},i-1)=$ Φ iff i=2n-3

v) If $D(L_n^2-\{2n\},i-1)\neq\Phi$, $D(L^2_{n-1},i-1)\neq\Phi$, $D(L^2_{n-1}-\{2n-2\},i-1)\neq\Phi$, $D(L^2_{n-2},i-1)\neq\Phi$, $D(L^2_{n-2}-\{2n-4\},i-1)\neq\Phi$ and $D(L^2_{n-3},i-1)=$ $D(L^2_{n-4},i-1)=\Phi$ iff i=2n-4.

Proof:

i) (⇒) Since $D(L^2_{n-\{2n\}},i-1)\neq\Phi$ and $D(L^2_{n-1},i-1)=$ $D(L^2_{n-1}-\{2n-2\},i-1)=D(L^2_{n-2},i-1)=\Phi$, $D(L^2_{n-2}-\{2n-4\},i-1)=D(L^2_{n-3},i-1)=$ $D(L^2_{n-4},i-1)=\Phi$, by lemma 2.2(iii),(iv)

We have

$\left\lfloor \frac{n}{4}\right\rfloor+1\le i-1\le 2n-1$,

$i-1\prec\left\lfloor \frac{n-1}{4}\right\rfloor+1$ or $i-1\succ 2n$-2, $i-1\prec\left\lfloor \frac{n-1}{4}\right\rfloor+1$ or $i-1\succ 2n$-3, $i-1\prec\left\lfloor \frac{n-2}{4}\right\rfloor+1$ or $i-1\succ 2n$-4, $i-1\prec\left\lfloor \frac{n-2}{4}\right\rfloor+1$ or $i-1\succ 2n$-5, $i-1\prec\left\lfloor \frac{n-3}{4}\right\rfloor+1$ or $i-1\succ 2n$-6 and $i-1\prec\left\lfloor \frac{n-4}{4}\right\rfloor+1$ or $i-1\succ 2n$-8.

Thus, from the above inequalities, we have $\left\lfloor \frac{n}{4}\right\rfloor+1\le i-1\le 2n-1$ and $i-1\prec\left\lfloor \frac{n-4}{4}\right\rfloor+1$ or $i-1\succ 2n-2$.

Since $i-1\ge\left\lfloor \frac{n}{4}\right\rfloor+1$, $i-1\not\subset\left\lfloor \frac{n-4}{4}\right\rfloor+1$.

Therefore, $i-1\succ 2n-2$. Therefore $i-1\ge 2n-1$.
But $i-1\le 2n-1$. Therefore $i-1=2n-1$.
Therefore i=2n.
Conversely,
Assume i=2n

Therefore, $D(L_n^2-\{2n\},i-1)=D(L_n^2-\{2n\},2n-1)\neq\Phi$;

$D(L^2_{n-1},i-1)=D(L^2_{n-1},2n-1)=\Phi$, since $2n-1\succ 2n-2$;

$D(L^2_{n-1}-\{2n-2\},i-1)=D(L^2_{n-1}-\{2n-2\},2n-1)=\Phi$, since $2n-1\succ 2n-3$; $D(L^2_{n-2},i-1)=D(L^2_{n-2},2n-1)=\Phi$, since $2n-1\succ 2n-4$;

$D(L^2_{n-2}-\{2n-4\},i-1)=D(L_{n-2}-\{2n-4\},2n-1)=\Phi$, since $2n-1\succ 2n-5$;

$D(L^2_{n-3},i-1)=D(L^2_{n-3},2n-1)=\Phi$, since $2n-1\succ 2n-6$;

$D(L^2_{n-4},i-1)=D(L^2_{n-4},2n-1)=\Phi$, since $2n-1\succ 2n-8$.

ii) (⇒) If $D(L_n^2-\{2n\},i-1)\neq\Phi$, $D(L^2_{n-1},i-1)\neq\Phi$ and $D(L^2_{n-1}-\{2n-2\},i-1)=D(L^2_{n-2},i-1)=\Phi$, $D(L^2_{n-2}-\{2n-4\},i-1)=D(L^2_{n-3},i-1)=$ $D(L^2_{n-4},i-1)=\Phi$, by lemma 2.2(iii),(iv)

$\left\lfloor \frac{n}{4}\right\rfloor+1\le i-1\le 2n-1$, $\left\lfloor \frac{n-1}{4}\right\rfloor+1\le i-1\le 2n-$2 and

$i-1\prec\left\lfloor \frac{n-1}{4}\right\rfloor+1$ or $i-1\succ 2n$-3, $i-1\prec\left\lfloor \frac{n-2}{4}\right\rfloor+1$ or $i-1\succ 2n$-4, $i-1\prec\left\lfloor \frac{n-2}{4}\right\rfloor+1$ or $i-1\succ 2n$-5,

$i-1\prec\left\lfloor \frac{n-3}{4}\right\rfloor+1$ or $i-1\succ 2n$-6,

$i-1\prec\left\lfloor \frac{n-4}{4}\right\rfloor+1$ or $i-1\succ 2n$-8.

Thus, from above inequalities, we have

$i-1\prec\left\lfloor \frac{n-4}{4}\right\rfloor+1$ or $i-1\succ 2n$-3.

Since $i-1\ge\left\lfloor \frac{n}{4}\right\rfloor+1$, $i-1\not\subset\left\lfloor \frac{n-4}{4}\right\rfloor+1$. Therefore, $i-1\succ 2n$-3.

Therefore, $i-1\ge 2n$-2.
But $i-1\le 2n-2$.
Therefore, $i-1=2n-2$.
Therefore i=2n-1.
(⇐)Conversely,
Assume i=2n-1

Therefore, $D(L_n^2-\{2n\},i-1)=D(L^2_{n-\{2n\}},2n-1)\neq\Phi$

$D(L^2_{n-1},i-1)=D(L^2_{n-1},2n-1)\neq\Phi$,

$D(L^2_{n-1}-\{2n-2\},i-1)=D(L^2_{(n-1)-\{2n-2\}},2n-1)=\Phi$, since $2n-2\succ 2n-3$ $D(L^2_{n-2},i-1)=$ $D(L^2_{n-2},2n-1)=\Phi$, since $2n-2\succ 2n-4$

$D(L^2_{n-2}-\{2n-4\},i-1)=\Phi$, since $2n-2\succ 2n-5$

$D(L^2_{n-3},i-1)=D(L^2_{n-3},2n-1)=\Phi$, since $2n-2\succ 2n-6$

$D(L^2_{n-4}, i-1) = D(L^2_{n-4}, 2n-1) = \Phi$,since
$2n-2 \succ 2n-8$.

iii) $(\Rightarrow)$ If $D(L^2_n - \{2n\}, i-1) \neq \Phi$, $D(L^2_{n-1}, i-1) \neq \Phi$,
$D(L^2_{n-1} - \{2n-2\}, i-1) \neq \Phi$
and $D(L^2_{n-2}, i-1) = \Phi$,
$D(L^2_{n-2} - \{2n-4\}, i-1) = D(L^2_{n-3}, i-1) =$
$D(L^2_{n-4}, i-1) = \Phi$, by lemma 2.2(iii),(iv)

$\left\lfloor \frac{n}{4} \right\rfloor + 1 \le i-1 \le 2n-1$, $\left\lfloor \frac{n-1}{4} \right\rfloor + 1 \le i-1 \le 2n-2$,
$\left\lfloor \frac{n-1}{4} \right\rfloor + 1 \le i-1 \le 2n-3$ (1)

$$\left.\begin{array}{l} i-1 \prec \left\lfloor \frac{n-2}{4} \right\rfloor + 1 \, or \, i-1 \succ 2n-4, \\ i-1 \prec \left\lfloor \frac{n-2}{4} \right\rfloor + 1 \, or \, i-1 \succ 2n-5, \\ i-1 \prec \left\lfloor \frac{n-3}{4} \right\rfloor + 1 \, or \, i-1 \succ 2n-6, \\ i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1 \, or \, i-1 \succ 2n-8 \end{array}\right\} \quad (2)$$

From the above inequalities (2) we have $i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or
$i-1 \succ 2n$-4

Since $i-1 \ge \left\lfloor \frac{n}{4} \right\rfloor + 1, i-1 \not\subset \left\lfloor \frac{n-4}{4} \right\rfloor + 1$.Therefore
$i-1 \succ 2n$-4.
Therefore, $i-1 \ge 2n-3$.But $i-1 \le 2n-3$.
Therefore, $i-1 = 2n-3$.
Therefore i=2n-2.
Converse is obious
The proof is as above

iv) $(\Rightarrow)$If $D(L^2_n - \{2n\}, i-1) \neq \Phi$ $D(L^2_{n-1}, i-1) \neq \Phi$,
$D(L^2_{n-1} - \{2n-2\}, i-1) \neq \Phi$,
$D(L^2_{n-2}, i-1) \neq \Phi$ and $D(L^2_{n-2} - \{2n-4\}, i-1) =$
$D(L^2_{n-3}, i-1) = D(L^2_{n-4}, i-1) = \Phi$

$\left\lfloor \frac{n}{4} \right\rfloor + 1 \le i-1 \le 2n-1$, $\left\lfloor \frac{n-1}{4} \right\rfloor + 1 \le i-1 \le 2n-2$,
$\left\lfloor \frac{n-1}{4} \right\rfloor + 1 \le i-1 \le 2n-3$ $\left\lfloor \frac{n-2}{4} \right\rfloor + 1 \le i-1 \le 2n-4$
(1)

$$\left.\begin{array}{l} i-1 \prec \left\lfloor \frac{n-2}{4} \right\rfloor + 1 \, or \, i-1 \succ 2n-5 \\ i-1 \prec \left\lfloor \frac{n-3}{4} \right\rfloor + 1 \, or \, i-1 \succ 2n-6 \\ i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1 \, or \, i-1 \succ 2n-8 \end{array}\right\}$$
(2)

From the above inequalities (2) we have $i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or
$i-1 \succ 2n$-5.

Since $i-1 \ge \left\lfloor \frac{n}{4} \right\rfloor + 1, i-1 \not\subset \left\lfloor \frac{n-4}{4} \right\rfloor + 1$.Therefore
$i-1 \succ 2n$-5.
Therefore, $i-1 \ge 2n-4$.But $i-1 \le 2n-4$.
Therefore, $i-1 = 2n-4$.
Therefore, i=2n-3.
Converse is obvious.
The proof is as above

v) $(\Rightarrow)$If $D(L^2_n - \{2n\}, i-1) \neq \Phi$, $D(L^2_{n-1}, i-1) \neq$
Φ, $D(L^2_{n-1} - \{2n-2\}, i-1) \neq \Phi$, $D(L^2_{n-2}, i-1)$
$\neq \Phi$
$D(L^2_{n-2} - \{2n-4\}, i-1) \neq \Phi$ and $D(L^2_{n-3}, i-1)$
$= D(L^2_{n-4}, i-1) = \Phi$

$\left\lfloor \frac{n}{4} \right\rfloor + 1 \le i-1 \le 2n-1$, $\left\lfloor \frac{n-1}{4} \right\rfloor + 1 \le i-1 \le 2n-2$,
$\left\lfloor \frac{n-1}{4} \right\rfloor + 1 \le i-1 \le 2n-3$ $\left\lfloor \frac{n-2}{4} \right\rfloor + 1 \le i-1 \le 2n-4$,
$\left\lfloor \frac{n-2}{4} \right\rfloor + 1 \le i-1 \le 2n-5$ and

$i-1 \prec \left\lfloor \frac{n-3}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-6, $i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or
$i-1 \succ 2n$-8.

From the above inequalities we have $i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or
$i-1 \succ 2n$-6.

Since $i-1 \ge \left\lfloor \frac{n}{4} \right\rfloor + 1, i-1 \not\subset \left\lfloor \frac{n-4}{4} \right\rfloor + 1$.Therefore
$i-1 \succ 2n$-6.
Therefore , $i-1 \ge 2n-5$.But $i-1 \le 2n-5$.
Therefore, $i-1 = 2n-5$.There fore i=2n-4.

Theorem2.5

i) If $D(L_n^2-\{2n\},i-1)\neq\Phi$ and $D(L_{n-1}^2,i-1)=D(L^2{}_{n-1}-\{2n-2\},i-1)=\Phi$,
$D(L_{n-2}^2,i-1)=\Phi$, $D(L^2{}_{n-2}-\{2n-4\},i-1)=D(L_{n-3}^2,i-1)=\Phi$,
$D(L_{n-4}^2,i-1)=\Phi$, then
$D(L_n^2,i)=\{X\cup\{2n\}/X\in D(L_n^2-\{2n\},i-1)\}$.

ii) If $D(L_n^2-\{2n\},i-1)\neq\Phi$, $D(L_{n-1}^2,i-1)\neq\Phi$ and $D(L^2{}_{n-1}-\{2n-2\},i-1)=D(L_{n-2}^2,i-1)=\Phi$,
$D(L^2{}_{n-2}-\{2n-4\},i-1)=D(L_{n-3}^2,i-1)=D(L_{n-4}^2,i-1)=\Phi$ then

$$D(L_n^2,i)=\{X_1\cup\{2n\}/X_1\in D(L_n^2-\{2n\},i-1)\}\cup\{X_2\cup\{2n-1\}/X_2\in D(L_{n-1}^2,i-1)\}$$

iii) If $D(L_n^2-\{2n\},i-1)\neq\Phi$ $D(L_{n-1}^2,i-1)\neq\Phi$ $D(L^2{}_{n-1}-\{2n-2\},i-1)\neq\Phi$ and $D(L_{n-2}^2,i-1)=\Phi$,
$D(L^2{}_{n-2}-\{2n-4\},i-1)=D(L_{n-3}^2,i-1)=D(L_{n-4}^2,i-1)=\Phi$, then

$$D(L_n^2,i)=\{X_1\cup\{2n\}/X_1\in D(L_n^2-\{2n\},i-1)\}\cup\{X_2\cup\{2n-1\}/X_2\in D(L_{n-1}^2,i-1)\}\cup\{X_3\cup\{2n-2\}/X_3\in D(L_{(n-1)}^2-\{2n-2\},i-1)\}$$

iv) If $D(L_n^2-\{2n\},i-1)\neq\Phi$, $D(L_{n-1}^2,i-1)\neq\Phi$, $D(L^2{}_{n-1}-\{2n-2\},i-1)\neq\Phi$,
$D(L_{n-2}^2,i-1)\neq\Phi$ and $D(L^2{}_{n-2}-\{2n-4\},i-1)=\Phi$
$D(L_{n-3}^2,i-1)=D(L_{n-4}^2,i-1)=\Phi$, then

$$D(L_n^2,i)=\{X_1\cup\{2n\}/X_1\in D(L_n^2-\{2n\},i-1)\}\cup\{X_2\cup\{2n-1\}/X_2\in D(L_{n-1}^2,i-1)\}\cup\{X_3\cup\{2n-2\}/X_3\in D(L_{(n-1)}^2-\{2n-2\},i-1)\}\cup\{X_4\cup\{2n-3\}/X_4\in D(L_{n-2}^2,i-1)\}$$

v) If $D(L_n^2-\{2n\},i-1)\neq\Phi$ $D(L_{n-1}^2,i-1)\neq\Phi$, $D(L^2{}_{n-1}-\{2n-2\},i-1)\neq\Phi$, $D(L_{n-2}^2,i-1)\neq\Phi$
$D(L^2{}_{n-2}-\{2n-4\},i-1)\neq\Phi$ and $D(L_{n-3}^2,i-1)=D(L_{n-4}^2,i-1)=\Phi$, then

$$D(L_n^2,i)=\{X_1\cup\{2n\}/X_1\in D(L_n^2-\{2n\},i-1)\}\cup\{X_2\cup\{2n-1\}/X_2\in D(L_{n-1}^2,i-1)\}\cup\{X_3\cup\{2n-2\}/X_3\in D(L_{(n-1)}^2-\{2n-2\},i-1)\}\cup\{X_4\cup\{2n-3\}/X_4\in D(L_{n-2}^2,i-1)\}\cup\{X_5\cup\{2n-4\}/X_5\in D(L_{(n-2)}^2-\{2n-4\},i-1)\}$$

vi) If $D(L_n^2-\{2n\},i-1)\neq\Phi$, $D(L_{n-1}^2,i-1)\neq\Phi$, $D(L^2{}_{n-1}-\{2n-2\},i-1)\neq\Phi$, $D(L_{n-2}^2,i-1)\neq\Phi$
$D(L^2{}_{n-2}-\{2n-4\},i-1)\neq\Phi$ $D(L_{n-3}^2,i-1)\neq\Phi$ and $D(L_{n-4}^2,i-1)=\Phi$ then

$$D(L_n^2,i)=\{X_1\cup\{2n\}/X_1\in D(L_n^2-\{2n\},i-1)\}\cup\{X_2\cup\{2n-1\}/X_2\in D(L_{n-1}^2,i-1)\}\cup\{X_3\cup\{2n-2\}/X_3\in D(L_{(n-1)}^2-\{2n-2\},i-1)\}\cup\{X_4\cup\{2n-3\}/X_4\in D(L_{n-2}^2,i-1)\}\cup\{X_5\cup\{2n-4\}/X_5\in D(L_{(n-2)}^2-\{2n-4\},i-1)\}\cup\{X_6\cup\{2n-5\}/X_6\in D(L_{n-3}^2,i-1)\}$$

vii) If $D(L_n^2-\{2n\},i-1)\neq\Phi$ $D(L_{n-1}^2,i-1)\neq\Phi$, $D(L^2{}_{n-1}-\{2n-2\},i-1)\neq\Phi$, $D(L_{n-2}^2,i-1)\neq\Phi$
$D(L^2{}_{n-2}-\{2n-4\},i-1)\neq\Phi$ $D(L_{n-3}^2,i-1)\neq\Phi$ and $D(L_{n-4}^2,i-1)\neq\Phi$ then

$$D(L_n^2,i)=\{X_1\cup\{2n\}/X_1\in D(L_n^2-\{2n\},i-1)\}\cup\{X_2\cup\{2n-1\}/X_2\in D(L_{n-1}^2,i-1)\}\cup\{X_3\cup\{2n-2\}/X_3\in D(L_{(n-1)}^2-\{2n-2\},i-1)\}\cup\{X_4\cup\{2n-3\}/X_4\in D(L_{n-2}^2,i-1)\}\cup\{X_5\cup\{2n-4\}/X_5\in D(L_{(n-2)}^2-\{2n-4\},i-1)\}\cup\{X_6\cup\{2n-5\}/X_6\in D(L_{n-3}^2,i-1)\}\cup\{X_7-\{2n-6\}\cup\{2n-1,2n-4\}/X_7\in D(L_{n-4}^2,i-1)\}$$

Proof:

i) By theorem 3(i),i=2n.Since in this case $D(L_n^2,i)=\{[2n]\}$ and $D(L_{n-\{2n\}}^2,i-1)=\{[2n-1]\}$,then we have the result.

ii) By theorem 3(ii),i=2n-1.Since in this case

$D(L_n^2,i)=\{[2n]-\{x\}/x\in[2n]\}$,

$D(L_{n-\{2n\}}^2,i-1)=\{[2n-1]-\{x\}/x\in[2n-1]\}$ and

$D(L_{n-1}^2,2n-2)=\{[2n-2]\}$ then

$$D(L_n^2,i)=\{X_1\cup\{2n\}/X_1\in D(L_n^2-\{2n\},i-1)\}\cup \{X_2\cup\{2n-1\}/X_2\in D(L_{n-1}^2,i-1)\}$$

iii) Let

$Y_1=\{X_1\cup\{2n\}/X_1\in D(L_n^2-\{2n\},i-1)\}$

$Y_2=\{X_2\cup\{2n-1\}/X_2\in D(L_{n-1}^2,i-1)\}$

$Y_3=\{X_3\cup\{2n-2\}/X_3\in D(L_{(n-1)}^2-\{2n-2\},i-1)\}$

Obviously $Y_1\cup Y_2\cup Y_3\subseteq D(L_n^2,i)$

Now, let $Y\in D(L_n^2,i)$ .If $2n\in Y$,then we can write

$Y=X_1\cup\{2n\}$, *forsome* $X_1\in D(L_n^2-\{2n\},i)$

,that is $Y\in Y_1$.

If 2n $\notin Y$ and 2n-1 $\in Y$,then we can write

$Y=X_2\cup\{2n-1\}$, *forsome* $X_2\in D(L_{n-1}^2,i-1)$

that is $Y\in Y_2$ .Now suppose that 2n-2 $\in Y$,2n $\notin Y$ and 2n-1 $\notin Y$,then we can write

$Y=X_3\cup\{2n-2\}$, *forsome* $X_3\in D(L_{(n-1)}^2-\{2n-2\},i-1)$

,

that is Y $\in Y_3$. Therefore we have proved

$D(L_n^2,i)\subseteq Y_1\cup Y_2\cup Y_3$.

Therefore we have

$$D(L_n^2,i)=\{X_1\cup\{2n\}/X_1\in D(L_n^2-\{2n\},i-1)\}\cup \{X_2\cup\{2n-1\}/X_2\in D(L_{n-1}^2,i-1)\}\cup \{X_3\cup\{2n-2\}/X_3\in D(L_{(n-1)}^2-\{2n-2\},i-1)\}$$

iv) Let

$Y_1=\{X_1\cup\{2n\}/X_1\in D(L_n^2-\{2n\},i-1)\}$

$Y_2=\{X_2\cup\{2n-1\}/X_2\in D(L_{n-1}^2,i-1)\}$

$Y_3=\{X_3\cup\{2n-2\}/X_3\in D(L_{(n-1)}^2-\{2n-2\},i-1)\}$

$Y_4=\{X_4\cup\{2n-3\}/X_4\in D(L_{n-2}^2,i-1)\}$

Obviously $Y_1\cup Y_2\cup Y_3\cup Y_4\subseteq D(L_n^2,i)$

(2.1)

Now, let $Y\in D(L_n^2,i)$ .If $2n\in Y$,then we can write

$Y=X_1\cup\{2n\}$ *forsome* $X_1\in D(L_n^2-\{2n\},i)$,that is $Y\in Y_1$.

If 2n $\notin Y$ and 2n-1 $\in Y$,then we can write

$Y=X_2\cup\{2n-1\}$, *forsome* $X_2\in D(L_{n-1}^2,i-1)$

that is $Y\in Y_2$.

Now suppose that 2n-2 $\in Y$,2n $\notin Y$ and 2n-1 $\notin Y$,then we can write

$Y=X_3\cup\{2n-2\}$, *forsome* $X_3\in D(L_{(n-1)}^2-\{2n-2\},i-1)$

,that is Y $\in Y_3$.

Now suppose that 2n-5 $\in Y$,2n-3 $\notin Y$ and 2n-4 $\notin Y$,we can write

$Y=X_4\cup\{2n-3\}$, *forsome* $X_4\in D(L_{n-2}^2,i-1)$

,that is Y $\in Y_4$.

Therefore we have proved

$D(L_n^2,i)\subseteq Y_1\cup Y_2\cup Y_3\cup Y_4$.

(2.2)

Hence, from (2.1) and (2.2)

$$D(L_n^2,i)=\{X_1\cup\{2n\}/X_1\in D(L_n^2-\{2n\},i-1)\}\cup \{X_2\cup\{2n-1\}/X_2\in D(L_{n-1}^2,i-1)\}\cup \{X_3\cup\{2n-2\}/X_3\in D(L_{(n-1)}^2-\{2n-2\},i-1)\}\cup \{X_4\cup\{2n-3\}/X_4\in D(L_{n-2}^2,i-1)\}$$

v) Let the

$Y_1=\{X_1\cup\{2n\}/X_1\in D(L_n^2-\{2n\},i-1)\}$

$Y_2=\{X_2\cup\{2n-1\}/X_2\in D(L_{n-1}^2,i-1)\}$

$Y_3=\{X_3\cup\{2n-2\}/X_3\in D(L_{(n-1)}^2-\{2n-2\},i-1)\}$

$Y_4=\{X_4\cup\{2n-3\}/X_4\in D(L_{n-2}^2,i-1)\}$

$Y_5=\{X_5\cup\{2n-4\}/X_5\in D(L_{n-2}^2-\{2n-4\},i-1)\}$

Obviously $Y_1\cup Y_2\cup Y_3\cup Y_4\cup Y_5\subseteq D(L_n^2,i)$

(2.3)

Now, let $Y\in D(L_n^2,i)$ .If $2n\in Y$,then we can write

$Y=X_1\cup\{2n\}$, *forsome* $X_1\in D(L_{n-\{2n\}}^2,i)$,that is $Y\in Y_1$.

If 2n $\notin Y$ and 2n-1 $\in Y$,then we can write

$Y=X_2\cup\{2n-1\}$, *forsome*$X_2\in D(L_{n-1}^2,i-1)$

that is $Y\in Y_2$.

Now suppose that 2n-2 $\in Y$,2n $\notin Y$ and 2n-1 $\notin Y$,then we can write

$Y=X_3\cup\{2n-2\}$, *forsome* $X_3\in D(L_{(n-1)}^2-\{2n-2\},i-1)$

,that is Y $\in Y_3$.

Suppose that 2n-3 $\in Y$, 2n, 2n-1,2n-2 $\notin Y$ then we can write

$$Y = X_4 \cup \{2n-3\},\ for some\ X_4 \in D(L_{n-2}^2, i-1).$$

If 2n-3 $\notin Y$,2n-4 $\in Y$ and any one of 2n,2n-1,2n-2 $\in Y$,then we can write

$$Y = X_5 - \{2n-3\} \cup \{2n-4, k, k\ is\ any\ on\ of\ 2n, 2n-1, 2n-2\},$$

$$for some\ X_5 \in D(L_{(n-2)}^2 - \{2n-4\}, i-1)$$

$$D(L_n^2, i) \subseteq Y_1 \cup Y_2 \cup Y_3 \cup Y_4 \cup Y_5$$

(2.4)

Therefore we have proved

$$D(L_n^2, i) = \{X_1 \cup \{2n\} / X_1 \in D(L_n^2 - \{2n\}, i-1)\} \cup$$
$$\{X_2 \cup \{2n-1\} / X_2 \in D(L_{n-1}^2, i-1)\} \cup$$
$$\{X_3 \cup \{2n-2\} / X_3 \in D(L_{(n-1)}^2 - \{2n-2\}, i-1)\} \cup$$
$$\{X_4 \cup \{2n-3\} / X_4 \in D(L_{n-2}^2, i-1)\} \cup$$
$$\{X_5 \cup \{2n-4\} / X_5 \in D(L_{(n-2)}^2 - \{2n-4\}, i-1)\}$$

vi) Let the

$$Y_1 = \{X_1 \cup \{2n\} / X_1 \in D(L_n^2 - \{2n\}, i-1)\}$$
$$Y_2 = \{X_2 \cup \{2n-1\} / X_2 \in D(L_{n-1}^2, i-1)\}$$
$$Y_3 = \{X_3 \cup \{2n-2\} / X_3 \in D(L_{(n-1)}^2 - \{2n-2\}, i-1)\}$$
$$Y_4 = \{X_4 \cup \{2n-3\} / X_4 \in D(L_{n-2}^2, i-1)\}$$
$$Y_5 = \{X_5 \cup \{2n-4\} / X_5 \in D(L_{n-2}^2 - \{2n-4\}, i-1)\}$$
$$Y_6 = \{X_6 \cup \{2n-5\} / X_6 \in D(L_{n-3}^2, i-1)\}$$

Obviously $Y_1 \cup Y_2 \cup Y_3 \cup Y_4 \cup Y \cup Y_6 \subseteq D(L_n^2, i)$ (2.5)

Now, let $Y \in D(L_n^2, i)$ .If $2n \in Y$,then we can write

$$Y = X_1 \cup \{2n\},\ for some\ X_1 \in D(L_{n-\{2n\}}^2, i)$$

,that is $Y \in Y_1$.

If 2n $\notin Y$ and 2n-1 $\in Y$,then we can write

$$Y = X_2 \cup \{2n-1\},\ for some X_2 \in D(L_{n-1}^2, i-1)$$

that is $Y \in Y_2$.

Now suppose that 2n-2 $\in Y$,2n $\notin Y$ and 2n-1 $\notin Y$,then we can write

$$Y = X_3 \cup \{2n-2\},\ for some\ X_3 \in D(L_{(n-1)}^2 - \{2n-2\}, i-1)$$

,that is Y $\in Y_3$.

Suppose that 2n-3 $\in Y$, 2n, 2n-1,2n-2 $\notin Y$ then we can write

$$Y = X_4 \cup \{2n-3\},\ for some\ X_4 \in D(L_{n-2}^2, i-1)$$

.

If 2n-4 $\in Y$ 2n,2n-1,2n-2,2n-3, $\notin Y$ and then 2n-5 $\in Y$, then we can write

$$Y = X_5 \cup \{2n-4\},\ for some\ X_5 \in D(L_{(n-2)}^2 - \{2n-4\}, i-1)$$

.

If 2n-4 $\notin Y$ then exactly two of and any one of 2n,2n-1,2n-2.2n-3 $\in Y$,then we can write

$$Y = X_6 - \{2n-4\} \cup \{i, k\}\ i, k\ indicates\ the\ exactly\ two\ ele$$

$$for some\ X_6 \in D(L_{n-3}^2, i-1)$$

$$D(L_n^2, i) \subseteq Y_1 \cup Y_2 \cup Y_3 \cup Y_4 \cup Y_5 \cup Y_6$$

(2.6)

Therefore we have proved

$$D(L_n^2, i) = \{X_1 \cup \{2n\} / X_1 \in D(L_n^2 - \{2n\}, i-1)\} \cup$$
$$\{X_2 \cup \{2n-1\} / X_2 \in D(L_{n-1}^2, i-1)\} \cup$$
$$\{X_3 \cup \{2n-2\} / X_3 \in D(L_{(n-1)}^2 - \{2n-2\}, i-1)\} \cup$$
$$\{X_4 \cup \{2n-3\} / X_4 \in D(L_{n-2}^2, i-1)\} \cup$$
$$\{X_5 \cup \{2n-4\} / X_5 \in D(L_{(n-2)}^2 - \{2n-4\}, i-1)\} \cup$$
$$\{X_6 \cup \{2n-5\} / X_6 \in D(L_{n-3}^2, i-1)\}$$

vii) Let the

$$Y_1 = \{X_1 \cup \{2n\} / X_1 \in D(L_n^2 - \{2n\}, i-1)\}$$
$$Y_2 = \{X_2 \cup \{2n-1\} / X_2 \in D(L_{n-1}^2, i-1)\}$$
$$Y_3 = \{X_3 \cup \{2n-2\} / X_3 \in D(L_{(n-1)}^2 - \{2n-2\}, i-1)\}$$
$$Y_4 = \{X_4 \cup \{2n-3\} / X_4 \in D(L_{n-2}^2, i-1)\}$$
$$Y_5 = \{X_5 \cup \{2n-4\} / X_5 \in D(L_{n-2}^2 - \{2n-4\}, i-1)\}$$
$$Y_6 = \{X_6 \cup \{2n-5\} / X_6 \in D(L_{n-3}^2, i-1)\}$$
$$Y_7 = \{X_7 - \{2n-6\} \cup \{2n-1, 2n-4\} / X_7 \in D(L_{n-4}^2, i-1)\}$$

Obviously

$$Y_1 \cup Y_2 \cup Y_3 \cup Y_4 \cup Y \cup Y_6 \cup Y_7 \subseteq D(L_n^2, i) \quad (2.7)$$

Now, let $Y \in D(L_n^2, i)$ .If $2n \in Y$,then we can write

$$Y = X_1 \cup \{2n, 2n-1, 2n-2, 2n-3\},\ for some\ X_1 \in D(L_{(n-2)}^2 - \{2n-4\}, i-1)$$

,that is $Y \in Y_1$.

If 2n $\notin Y$ and 2n-1 $\in Y$,then we can write

$$Y = X_2 \cup \{2n-2, 2n-3\},\ for some\ X_2 \in D(L_{n-3}^2, i-1)$$

that is $Y \in Y_2$

.Now suppose that 2n-2 $\in Y$,2n $\notin Y$ and 2n-1 $\notin Y$,then we can write

$$Y = X_3 \cup \{2n\},\ for some\ X_3 \in D(L_{n-4}^2, i-1), X_3\ is\ ends\ with\ n-1$$

,that is Y $\in Y_3$.

Suppose that 2n-3 $\in Y$, 2n, 2n-1,2n-2 $\notin Y$ then we can write

$Y = X_4 \cup \{2n-1\}, \, for some \, X_3 \in D(L^2_{n-4}, i-1), X_4 \, is \, ends \, with \, n$

.

If 2n-4 $\in Y$ 2n,2n-1,2n-2,2n-3, $\notin Y$ and then 2n-5 $\in Y$,then we can write

$Y = X_5 \cup \{2n-2\}, \, for some \, X_5 \in D(L^2_{n-4}, i-1), X_3 \, is \, ends \, with \, n-3$

.

If 2n-6 $\notin Y$ and exactly two of 2n,2n-1,2n-2,2n-3,2n-4,2n-5 $\in Y$, then we can write

$Y = X_7 - \{2n-6\} \cup \{i,k\} \, i,k \, indicates \, the \, exactly \, two \, elements \, of \, 2n, 2n-1,$

$for some \, X_7 \in D(L^2_{n-4}, i-1)$

Hence

$$D(L^2_n, i) \subseteq Y_1 \cup Y_2 \cup Y_3 \cup Y_4 \cup Y_5 \cup Y_6 \cup Y_7 \quad (2.8)$$

$$D(L^2_n, i) = \{X_1 \cup \{2n\} / X_1 \in D(L^2_n - \{2n\}, i-1)\} \cup$$
$$\{X_2 \cup \{2n-1\} / X_2 \in D(L^2_{n-1}, i-1)\} \cup$$
$$\{X_3 \cup \{2n-2\} / X_3 \in D(L^2_{(n-1)} - \{2n-2\}, i-1)\} \cup$$
$$\{X_4 \cup \{2n-3\} / X_4 \in D(L^2_{n-2}, i-1)\} \cup$$
$$\{X_5 \cup \{2n-4\} / X_5 \in D(L^2_{(n-2)} - \{2n-4\}, i-1)\} \cup$$
$$\{X_6 \cup \{2n-5\} / X_6 \in D(L^2_{n-3}, i-1)\} \cup$$
$$\{X_7 - \{2n-6\} \cup \{2n-1, 2n-4\} / X_7 \in D(L^2_{n-4}, i-1)\}$$

3 Domination sets of $D(L^2_n - \{2n\}, i)$

For the construction of $D(L^2_n - \{2n\}, i)$,we consider, $D(L^2_{n-1}, i-1)$, $D(L^2_{n-1} - \{2n-2\}, i-1)$, $D(L^2_{n-2}, i-1)$, $D(L^2_{n-2} - \{2n-4\}, i-1)$, $D(L^2_{n-3}, i-1)$, $D(L^2_{n-4}, i-1)$ and $D(L^2_{(n-4)} - \{2n-8\}, i-1)$.

Lemma 3.1

i) If $D(L^2_{n-1}, i-1) = D(L^2_{n-1} - \{2n-2\}, i-1) = D(L^2_{n-2}, i-1) = \Phi$

$D(L^2_{n-2} - \{2n-4\}, i-1) = \Phi$, $D(L^2_{n-3}, i-1) = D(L^2_{n-4}, i-1) = \Phi$

$D(L^2_{(n-4)} - \{2n-8\}, i-1) = \Phi$,then $D(L^2_{n-\{2n\}}, i) = \Phi$

ii) If $D(L^2_{n-1}, i-1) = D(L^2_{n-1} - \{2n-2\}, i-1) = D(L^2_{n-2}, i-1) = \Phi$

$D(L^2_{n-2} - \{2n-4\}, i-1) = \Phi$, $D(L^2_{n-3}, i-1) = \Phi$ and

$D(L^2_{(n-4)} - \{2n-8\}, i-1) \neq \Phi$ then $D(L^2_{n-\{2n\}}, i) \neq \Phi$

Proof

i) Since $D(L^2_{n-1}, i-1) = D(L^2_{n-1} - \{2n-2\}, i-1) = D(L^2_{n-2}, i-1) = \Phi$

$D(L^2_{n-2} - \{2n-4\}, i-1) = \Phi$, $D(L^2_{n-3}, i-1) = D(L^2_{n-4}, i-1) = \Phi$

$D(L^2_{(n-4)} - \{2n-8\}, i-1) = \Phi$, by lemma 2.2(iii),(iv),

$i-1 \prec \left\lfloor \frac{n-1}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-2, $i-1 \prec \left\lfloor \frac{n-1}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-3, $i-1 \prec \left\lfloor \frac{n-2}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-4, $i-1 \prec \left\lfloor \frac{n-2}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-5,

$i-1 \prec \left\lfloor \frac{n-3}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-6, $i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-8.

$i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-9

Therefore, we have $i \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 2$ or $i-1 \succ 2n$-2

That is $i \prec \left\lfloor \frac{n}{4} \right\rfloor + 1$ or i $\succ 2n$-1

Therefore, $D(L^2_n - \{2n\}, i) = \Phi$.

ii) If $D(L^2_{n-1}, i-1) = D(L^2_{n-1} - \{2n-2\}, i-1) = D(L^2_{n-2}, i-1) = \Phi$

$D(L^2_{n-2} - \{2n-4\}, i-1) = \Phi$, $D(L^2_{n-3}, i-1) = \Phi$ and $D(L^2_{(n-4)} - \{2n-8\}, i-1) \neq \Phi$, by lemma 2.2(iii),(iv),

$i-1 \prec \left\lfloor \frac{n-1}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-2, $i-1 \prec \left\lfloor \frac{n-1}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-3,

$i-1 \prec \left\lfloor \frac{n-2}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-4, $i-1 \prec \left\lfloor \frac{n-2}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-5,

$i-1 \prec \left\lfloor \frac{n-3}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-6,

and $\left\lfloor \frac{n-4}{4} \right\rfloor + 1 \leq i-1 \leq 2n$-7.

From the above inequalities, we have

$i-1 \prec \left\lfloor \frac{n-3}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-2 and $\left\lfloor \frac{n-4}{4} \right\rfloor + 1 \leq i-1 \leq 2n$-7.

$i-1 \leq 2n-8$, $i-1 \succ 2n-2$.

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Therefore, $i-1 \prec \left\lfloor \frac{n-3}{4} \right\rfloor +1$ and $\left\lfloor \frac{n-4}{4} \right\rfloor +1 \le i-1 \le 2n$-7

$\left\lfloor \frac{n-4}{4} \right\rfloor +1 \le i-1 \prec \left\lfloor \frac{n-3}{4} \right\rfloor +1$.

Therefore, $\left\lfloor \frac{n-4}{4} \right\rfloor +2 \le i \prec \left\lfloor \frac{n-3}{4} \right\rfloor +2$,

Since, $\left\lfloor \frac{n}{4} \right\rfloor +1 \le \left\lfloor \frac{n-4}{4} \right\rfloor +2$

We have, $i \ge \left\lfloor \frac{n}{4} \right\rfloor +1$

Therefore, $D(L^2_{n-\{2n\}}, i) \ne \Phi$.

Theorem 3.2

For every n, $D(L_n^2 - \{2n\}, i-1) \ne \Phi$

i) If $D(L^2_{n-1}, i-1) \ne \Phi$ and $D(L^2_{n-1} - \{2n-2\}, i-1) = D(L^2_{n-2}, i-1) = \Phi$

$D(L^2_{n-2} - \{2n-4\}, i-1) = \Phi$ $D(L^2_{n-3}, i-1) = \Phi$

$D(L^2_{n-4}, i-1) = D(L^2_{(n-4)} - \{2n-8\}, i-1) = \Phi$ iff i=2n.

ii) If $D(L^2_{n-1}, i-1) \ne \Phi$, $D(L^2_{n-1} - \{2n-2\}, i-1) \ne \Phi$ and

$D(L^2_{n-2}, i-1) = \Phi$, $D(L^2_{n-2} - \{2n-4\}, i-1) = D(L^2_{n-3}, i-1) = \Phi$

$D(L^2_{n-4}, i-1) = D(L^2_{(n-4)} - \{2n-8\}, i-1) = \Phi$ iff i=2n-1

iii) If $D(L^2_{n-1}, i-1) \ne \Phi$, $D(L^2_{n-1} - \{2n-2\}, i-1) \ne \Phi$,

$D(L^2_{n-2}, i-1) \ne \Phi$ and $D(L^2_{n-2} - \{2n-4\}, i-1) = \Phi$

$D(L^2_{n-3}, i-1) = \Phi$

$D(L^2_{n-4}, i-1) = D(L^2_{(n-4)} - \{2n-8\}, i-1) = \Phi$ iff i=2n-2

iv) If $D(L^2_{n-1}, i-1) \ne \Phi$, $D(L^2_{n-1} - \{2n-2\}, i-1) \ne \Phi$, $D(L^2_{n-2}, i-1) \ne \Phi$,

$D(L^2_{n-2} - \{2n-4\}, i-1) \ne \Phi$ and $D(L^2_{n-3}, i-1) = D(L^2_{n-4}, i-1) = \Phi$

$D(L^2_{(n-4)} - \{2n-8\}, i-1) = \Phi$ iff i=2n-3

v) If $D(L^2_{n-1}, i-1) \ne \Phi$, $D(L^2_{n-1} - \{2n-2\}, i-1) \ne \Phi$, $D(L^2_{n-2}, i-1) \ne \Phi$,

$D(L^2_{n-2} - \{2n-4\}, i-1) \ne \Phi$ $D(L^2_{n-3}, i-1) \ne \Phi$ and

$D(L^2_{n-4}, i-1) = \Phi$

$D(L^{*2}_{(n-4)} - \{2n-8\}, i-1) = \Phi$ iff i=2n-4.

Proof: $D(L_n^2 - \{2n\}, i-1) \ne \Phi$

i) $(\Rightarrow)$ $D(L^2_{n-1}, i-1) \ne \Phi$ and

$D(L^2_{n-1} - \{2n-2\}, i-1) = D(L^2_{n-2}, i-1) = \Phi$

$D(L^2_{n-2} - \{2n-4\}, i-1) = \Phi$, $D(L^2_{n-3}, i-1) = D(L^2_{n-4}, i-1) = \Phi$

$D(L^2_{(n-4)} - \{2n-8\}, i-1) = \Phi$, by lemma 2.2(iii),(iv),

$\left\lfloor \frac{n}{4} \right\rfloor +1 \le i-1 \le 2n-2$ and

$i-1 \prec \left\lfloor \frac{n-1}{4} \right\rfloor +1$ or $i-1 \succ 2n$-3, $i-1 \prec \left\lfloor \frac{n-2}{4} \right\rfloor +1$ or $i-1 \succ 2n$-5,

$i-1 \prec \left\lfloor \frac{n-2}{4} \right\rfloor +1$ or $i-1 \succ 2n$-4, $i-1 \prec \left\lfloor \frac{n-3}{4} \right\rfloor +1$ or $i-1 \succ 2n$-5,

$i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor +1$ or $i-1 \succ 2n$-6,

$i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor +1$ or $i-1 \succ 2n$-8.

Thus, from above inequalities, we have

$\left\lfloor \frac{n-1}{4} \right\rfloor +1 \le i-1 \le 2n-2$ and $i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor +1$ or $i-1 \succ 2n-2$.

Since $i-1 \ge \left\lfloor \frac{n-1}{4} \right\rfloor +1$, $i-1 \not\prec \left\lfloor \frac{n-4}{4} \right\rfloor +1$.

Therefore, $i-1 \succ 2n-2$. Therefore $i-1 \ge 2n-1$.

But $i-1 \le 2n-1$.

Therefore, $i-1 = 2n-1$.

Therefore i=2n.

Conversely,

Assume i=2n

Therefore, $D(L^2_n - \{2n\}, i-1) = D(L^2_n - \{2n\}, 2n-1) \ne \Phi$

$D(L^2_{n-1}, i-1) = D(L^2_{n-1}, 2n-1) = \Phi$,

$D(L^2_{n-1} - \{2n-2\}, i-1) = D(L^2_{(n-1)} - \{2n-2\}, 2n-1) = \Phi$, since $2n-1 \succ 2n-3$ $D(L^2_{n-2}, i-1) == D(L^2_{n-2}, 2n-1) = \Phi$, since $2n-1 \succ 2n-4$

$D(L^2_{n-2} - \{2n-4\}, i-1) = D(L^2_{n-2} - \{2n-4\}, i-1) = \Phi$, since $2n-1 \succ 2n-5$

$D(L^2_{n-3}, i-1) = D(L^2_{n-3}, 2n-1) = \Phi$,since
$2n-1 \succ 2n-6$
$D(L^2_{n-4}, i-1) = D(L^2_{n-4}, 2n-1) = \Phi$,since
$2n-1 \succ 2n-8$
$D(L^2_{(n-4)} - \{2n-8\}, i-1) = D(L^2_{(n-2)} - \{2n-8\}, 2n-1) = \Phi$,
since $2n-1 \succ 2n-9$.

ii) ($\Rightarrow$)Assume $D(L^2_{n-1}, i-1) \neq \Phi$, $D(L^2{}_n - \{2n\}, i-1) \neq \Phi$ and
$D(L^2_{n-2}, i-1) = D(L^{*2}_{(n-2)} - \{2n-4\}, i-1) = D(L^2_{n-3}, i-1) = D(L^2_{n-4}, i-1) = \Phi$
$D(L^2_{(n-4)} - \{2n-8\}, i-1) = \Phi$.

$\left\lfloor \frac{n-1}{4} \right\rfloor + 1 \le i-1 \le 2n-2$, $\left\lfloor \frac{n}{4} \right\rfloor + 1 \le i-1 \le 2n-1$ and

$i-1 \prec \left\lfloor \frac{n-2}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-3, $i-1 \prec \left\lfloor \frac{n-2}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-4,

$i-1 \prec \left\lfloor \frac{n-3}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-5, $i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-6,

$i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-8.

Thus, from above, inequalities, we have

$\left\lfloor \frac{n}{4} \right\rfloor + 1 \le i-1 \le 2n-2$ and $i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-3.

Since $i-1 \ge \left\lfloor \frac{n}{4} \right\rfloor + 1$, $i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$.

Therefore , $i-1 \succ 2n$-3.
Therefore, $i-1 \ge 2n-2$.
But $i-1 \le 2n-2$.
Therefore, $i-1 = 2n-2$.
Therefore i=2n-1.

Conversely,
Assume i=2n-1
Therefore, $D(L^2_{n-1}, i-1) = D(L^2_{n-1}, 2n-1) \neq \Phi$,
$D(L^2{}_{n-1} - \{2n-2\}, i-1) = D(L^2_{(n-1)} - \{2n-2\}, 2n-1) \neq \Phi$,
$D(L^2_{n-2}, i-1) == D(L^2_{n-2}, 2n-1) = \Phi$,since
$2n-2 \succ 2n-4$,
$D(L^2{}_{n-2} - \{2n-4\}, i-1) = D(L^2{}_{n-2} - \{2n-4\}, i-1) = \Phi$
,since $2n-2 \succ 2n-5$,
$D(L^2_{n-3}, i-1) = D(L^2_{n-3}, 2n-1) = \Phi$,since
$2n-2 \succ 2n-6$,
$D(L^2_{n-4}, i-1) = D(L^2_{n-4}, 2n-1) = \Phi$,since
$2n-2 \succ 2n-8$,
$D(L^2_{(n-4)} - \{2n-8\}, i-1) = D(L^2_{(n-2)} - \{2n-8\}, 2n-1) = \Phi$,
since $2n-2 \succ 2n-9$.

iii) ($\Rightarrow$)Assume $D(L^2_{n-1}, i-1) \neq \Phi$, $D(L^2{}_n - \{2n\}, i-1) \neq \Phi$, $D(L^2{}_{n-1} - \{2n-2\}, i-1) \neq \Phi$,
$D(L^2_{n-2}, i-1) \neq \Phi$ and
$D(L^2{}_{n-2} - \{2n-4\}, i-1) = \Phi$ $D(L^2_{n-3}, i-1) =$
$D(L^2_{n-4}, i-1) = D(L^2_{(n-4)} - \{2n-8\}, i-1) = \Phi$

$\left\lfloor \frac{n-1}{4} \right\rfloor + 1 \le i-1 \le 2n-1$, $\left\lfloor \frac{n-1}{4} \right\rfloor + 1 \le i-1 \le 2n-2$, $\left\lfloor \frac{n-2}{4} \right\rfloor + 1 \le i-1 \le 2n-3$

$i-1 \prec \left\lfloor \frac{n-2}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-4, $i-1 \prec \left\lfloor \frac{n-3}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-5,

$i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-6, $i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-7.

Thus, from above inequalities

$\left\lfloor \frac{n-1}{4} \right\rfloor + 1 \le i-1 \le 2n-3$ and $i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-4.

Since $i-1 \ge \left\lfloor \frac{n-1}{4} \right\rfloor + 1$, $i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$.

Therefore, $i-1 \succ 2n$-4.
Therefore, $i-1 \ge 2n-3$.
But, $i-1 \le 2n-3$.
Therefore, $i-1 = 2n-3$.
Therefore, i=2n-2.
Proof is similar to above.

iv) ($\Rightarrow$)If $D(L^2_{n-1}, i-1) \neq \Phi$, $D(L^2_{(n-1)-\{2n-2\}}, i-1) \neq \Phi$,
$D(L^2_{n-2}, i-1) \neq \Phi$,
$D(L^{*2}_{(n-2)-\{2n-4\}}, i-1) \neq \Phi$ and $D(L^2_{n-3}, i-1) =$
$D(L^2_{n-4}, i-1) = \Phi$
$D(L^{*2}_{(n-4)-\{2n-8\}}, i-1) = \Phi$

$\left\lfloor \frac{n-1}{4} \right\rfloor + 1 \le i-1 \le 2n-1$, $\left\lfloor \frac{n-1}{4} \right\rfloor + 1 \le i-1 \le 2n-2$,

$\left\lfloor \frac{n-2}{4} \right\rfloor + 1 \le i-1 \le 2n-3$

$\left\lfloor \frac{n-2}{4} \right\rfloor + 1 \le i-1 \le 2n-4$, $i-1 \prec \left\lfloor \frac{n-3}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-5,

$i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-6, $i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-8.

Thus, from above, inequalities

$\left\lfloor \frac{n-1}{4} \right\rfloor + 1 \le i-1 \le 2n-4$ and $i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-5.

Since $i-1 \ge \left\lfloor \frac{n-1}{4} \right\rfloor + 1$, $i-1 \not\subset \left\lfloor \frac{n-4}{4} \right\rfloor + 1$.

Therefore , $i-1 \succ 2n$-5.Therefore, $i-1 \ge 2n-4$.

But, $i-1 \le 2n-4$.

Therefore $i-1 = 2n-4$.

Therefore i=2n-3.

Proof is similar to above

v) ($\Rightarrow$)If $D(L_{n-1}^2, i-1) \ne \Phi$, $D(L^2_{(n-1)-\{2n-2\}}, i-1) \ne \Phi$,

$D(L_{n-2}^2, i-1) \ne \Phi$, $D(L^{*2}_{(n-2)-\{2n-4\}}, i-1) \ne \Phi$,

$D(L_{n-3}^2, i-1) \ne \Phi$ and

$D(L_{n-4}^2, i-1) = D(L^{*2}_{(n-4)-\{2n-8\}}, i-1) = \Phi$

$\left\lfloor \frac{n-1}{4} \right\rfloor + 1 \le i-1 \le 2n-1$, $\left\lfloor \frac{n-1}{4} \right\rfloor + 1 \le i-1 \le 2n-2$,

$\left\lfloor \frac{n-2}{4} \right\rfloor + 1 \le i-1 \le 2n-3$

$\left\lfloor \frac{n-2}{4} \right\rfloor + 1 \le i-1 \le 2n-4$, $\left\lfloor \frac{n-3}{4} \right\rfloor + 1 \le i-1 \le 2n-5$,

$i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-6, $i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-8.

Thus, from above, inequalities, we have

$\left\lfloor \frac{n-1}{4} \right\rfloor + 1 \le i-1 \le 2n-5$ and $i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$ or $i-1 \succ 2n$-6.

Since $i-1 \ge \left\lfloor \frac{n-1}{4} \right\rfloor + 1$, $i-1 \prec \left\lfloor \frac{n-4}{4} \right\rfloor + 1$.

Therefore, $i-1 \succ 2n$-6.Therefore, $i-1 \ge 2n-5$.

But, $i-1 \le 2n-5$.

Therefore, $i-1 = 2n-5$.

Therefore, i=2n-4.

Proof is similar to above.

Theorem 3.3

For every n

i) If $D(L_{n-1}^2, i-1) \ne \Phi$ and

$D(L_{(n-1)}^2 - \{2n-2\}, i-1) = D(L_{n-2}^2, i-1) = \Phi$

$D(L_{(n-2)}^2 - \{2n-4\}, i-1) = \Phi$ $D(L_{n-3}^2, i-1) =$

$D(L_{n-4}^2, i-1) = \Phi$

$D(L_{(n-4)}^2 - \{2n-8\}, i-1) = \Phi$ then

$D(L_n^2 - \{2n\}, i) = \{X \cup \{2n\} / X \in D(L_{n-1}^2, i-1)\}$

ii) If $D(L_{n-1}^2, i-1) \ne \Phi$, $D(L_{(n-1)}^2 - \{2n-2\}, i-1) \ne \Phi$ and

$D(L_{n-2}^2, i-1) = \Phi$ $D(L_{(n-2)}^2 - \{2n-4\}, i-1) = \Phi$

$D(L_{n-3}^2, i-1) = D(L_{n-4}^2, i-1) = D(L_{(n-4)}^2 - \{2n-8\}, i-1) = \Phi$

then

$D(L_n^2 - \{2n\}, i) = \{X_1 \cup \{2n\} / X_1 \in D(L_{n-1}^2, i-1)\} \cup$

$\{X_2 \cup \{2n-1\} / X_2 \in D(L_{(n-1)}^2 - \{2n-2\}, i-1)\}$

iii) If $D(L_{n-1}^2, i-1) \ne \Phi$, $D(L_{(n-1)}^2 - \{2n-2\}, i-1) \ne \Phi$, $D(L_{n-2}^2, i-1) \ne \Phi$ and

$D(L_{(n-2)}^2 - \{2n-4\}, i-1) = \Phi$ $D(L_{n-3}^2, i-1) =$

$D(L_{n-4}^2, i-1) = \Phi$

$D(L_{(n-4)}^2 - \{2n-8\}, i-1) = \Phi$ then

$D(L_n^2 - \{2n\}, i) = \{X_1 \cup \{2n\} / X_1 \in D(L_{n-1}^2, i-1)\} \cup$

$\{X_2 \cup \{2n-1\} / X_2 \in D(L_{(n-1)}^2 - \{2n-2\}, i-1)\} \cup$

$\{X_3 \cup \{2n-2\} / X_3 \in D(L_{n-2}^2, i-1)\}$

iv) If $D(L_{n-1}^2, i-1) \ne \Phi$, $D(L_{(n-1)}^2 - \{2n-2\}, i-1) \ne \Phi$,

$D(L_{n-2}^2, i-1) \ne \Phi$, $D(L_{(n-2)}^2 - \{2n-4\}, i-1) \ne \Phi$

and $D(L_{n-3}^2, i-1) = D(L_{n-4}^2, i-1) =$

$D(L_{(n-4)}^2 - \{2n-8\}, i-1) = \Phi$ then

$D(L_n^2 - \{2n\}, i) = \{X_1 \cup \{2n\} / X_1 \in D(L_{n-1}^2, i-1)\} \cup$

$\{X_2 \cup \{2n-1\} / X_2 \in D(L_{(n-1)}^2 - \{2n-2\}, i-1)\} \cup$

$\{X_3 \cup \{2n-2\} / X_3 \in D(L_{n-2}^2, i-1)\} \cup$

$\{X_4 \cup \{2n-3\} / X_4 \in D(L_{(n-2)}^2 - \{2n-4\}, i-1)\}$

v) If $D(L_{n-1}^2, i-1) \ne \Phi$, $D(L_{(n-1)}^2 - \{2n-2\}, i-1) \ne \Phi$,

$D(L_{n-2}^2, i-1) \ne \Phi$, $D(L_{(n-2)}^2 - \{2n-4\}, i-1) \ne \Phi$

$D(L_{n-3}^2, i-1) \ne \Phi$ and $D(L_{n-4}^2, i-1) =$

$D(L^2_{(n-4)}-\{2n-8\},i-1)=\Phi$ then

$$D(L^2_n-\{2n\},i)=\{X_1\cup\{2n\}/X_1\in D(L^2_{n-1},i-1)\}\cup$$
$$\{X_2\cup\{2n-1\}/X_2\in D(L^2_{(n-1)}-\{2n-2\},i-1)\}\cup$$
$$\{X_3\cup\{2n-2\}/X_3\in D(L^2_{n-2},i-1)\}\cup$$
$$\{X_4\cup\{2n-3\}/X_4\in D(L^2_{(n-2)}-\{2n-4\},i-1)\}\cup$$
$$\{X_5\cup\{2n-4\}/X_5\in D(L^2_{n-3},i-1)\}$$

Proof:

i) By theorem 3(i),i=2n.Since in this case $D(L^2_{n-\{2n\}},i)$ = $\{[2n]\}$ and $D(L^2_{n-1},i-1)=\{[2n-1]\}$,then we have the result.

$$D(L^2_n-\{2n\},i)=\{X\cup\{2n\}/X\in D(L^2_{n-1},i-1)\}$$

ii) By theorem 3(ii),i=2n-1.Since in this case $D(L^2_n-\{2n\},i)$ $=\{[2n]-\{x\}/x\in[2n]\}$, $D(L^2_{n-1},i-1)=$ $\{[2n-1]-\{x\}/x\in[2n-1]\}$ and $D(L^2_{(n-1)}-\{2n\},2n-2)=\{[2n-2]\}$ then

$$D(L^2_n-\{2n\},i)=\{X_1\cup\{2n\}/X_1\in D(L^2_{n-1},i-1)\}\cup$$
$$\{X_2\cup\{2n-1\}/X_2\in D(L^2_{(n-1)}-\{2n\},i-1)\}$$

iii) Let

$$Y_1=\{X_1\cup\{2n\}/X_1\in D(L^2_{n-1},i-1)\}$$
$$Y_2=\{X_2\cup\{2n-1\}/X_2\in D(L^2_{(n-1)}-\{2n-2\},i-1)\}$$
$$Y_3=\{X_3\cup\{2n-2\}/X_3\in D(L^2_{n-2},i-1)\}$$

Obviously $Y_1\cup Y_2\cup Y_3\subseteq D(L^2_n,i)$
(3.1)

Now, let $Y\in D(L^2_n,i)$ .If $2n\in Y$,then we can write $Y=X_1\cup\{2n\}$, $forsome\ X_1\in D(L^2_{n-1},i-1)$,that is $Y\in Y_1$.

If 2n $\notin Y$ and 2n-1 $\in Y$,then we can write $Y=X_2\cup\{2n-1\}$, $forsome X_2\in D(L^2_{n-1}-\{2n-2\},i-1)$ that is $Y\in Y_2$.

Now suppose that 2n-2 $\in Y$,2n $\notin Y$ and 2n-1 $\notin Y$,then we can write $Y=X_3\cup\{2n-3\}$, $forsome\ X_3\in D(L^2_{(n-2)},i-1)$,that is Y $\in Y_3$.

Therefore we have proved

$$D(L^2_n-\{2n\},i)\subseteq Y_1\cup Y_2\cup Y_3.$$

(3.2)

Therefore we have

$$D(L^2_n-\{2n\},i)=\{X_1\cup\{2n\}/X_1\in D(L^2_{n-1},i-1)\}\cup$$
$$\{X_2\cup\{2n-1\}/X_2\in D(L^2_{(n-1)}-\{2n-2\},i-1)\}\cup$$
$$\{X_3\cup\{2n-2\}/X_3\in D(L^2_{n-2},i-1)\}$$

iv) Let

$$Y_1=\{X_1\cup\{2n\}/X_1\in D(L^2_{n-1},i-1)\}$$
$$Y_2=\{X_2\cup\{2n-1\}/X_2\in D(L^2_{(n-1)}-\{2n\},i-1)\}$$
$$Y_3=\{X_3\cup\{2n-2\}/X_3\in D(L^2_{n-2},i-1)\}$$
$$Y_4=\{X_4\cup\{2n-3\}/X_4\in D(L^2_{(n-1)}-\{2n-2\},i-1)\}$$

Obviously $Y_1\cup Y_2\cup Y_3\cup Y_4\subseteq D(L^2_n-\{2n\},i)$
(3.3)

Now, let $Y\in D(L^2_n,i)$ .If $2n\in Y$,then we can write $Y=X_1\cup\{2n\}$, $forsome\ X_1\in D(L^2_{n-1},i-1)$,that is $Y\in Y_1$.

If 2n $\notin Y$ and 2n-1 $\in Y$,then we can write $Y=X_2\cup\{2n-1\}$, $forsome X_2\in D(L^2_{n-1}-\{2n-2\},i-1)$ that is $Y\in Y_2$.

Now suppose that 2n-2 $\in Y$,2n $\notin Y$ and 2n-1 $\notin Y$,then we can write $Y=X_3\cup\{2n-3\}$, $forsome\ X_3\in D(L^2_{(n-2)},i-1)$,that is Y $\in Y_3$.

Now suppose that 2n-5 $\in Y$,2n-3 $\notin Y$ and 2n-4 $\notin Y$,we can write $Y=X_4\cup\{2n-3\}$, $forsome\ X_4\in D(L^2_{n-1}-\{2n-2\},i-1)$,that is Y $\in Y_4$.

Therefore, we have proved

$$D(L^2_n-\{2n\},i)\subseteq Y_1\cup Y_2\cup Y_3\cup Y_4$$

(3.4)
Hence

$$D(L^2_n-\{2n\},i)=\{X_1\cup\{2n\}/X_1\in D(L^2_{n-1},i-1)\}\cup$$
$$\{X_2\cup\{2n-1\}/X_2\in D(L^2_{(n-1)}-\{2n-2\},i-1)\}\cup$$
$$\{X_3\cup\{2n-2\}/X_3\in D(L^2_{n-2},i-1)\}\cup$$
$$\{X_4\cup\{2n-3\}/X_4\in D(L^2_{(n-2)}-\{2n-4\},i-1)\}$$

v) Let

$$Y_1 = \{X_1 \cup \{2n\} / X_1 \in D(L^2_{n-1}, i-1)\}$$
$$Y_2 = \{X_2 \cup \{2n-1\} / X_2 \in D(L^2_{(n-1)} - \{2n\}, i-1)\}$$
$$Y_3 = \{X_3 \cup \{2n-2\} / X_3 \in D(L^2_{n-2}, i-1)\}$$
$$Y_4 = \{X_4 \cup \{2n-3\} / X_4 \in D(L^2_{(n-1)} - \{2n-2\}, i-1)\}$$
$$Y_5 = \{X_5 \cup \{2n-2\} / X_5 \in D(L^2_{n-3}, i-1)\}$$

Obviously $Y_1 \cup Y_2 \cup Y_3 \cup Y_4 \cup Y_5 \subseteq D(L^2_{n-\{2n\}}, i)$
(3.5)

Now, let $Y \in D(L^2_n, i)$ .If $2n \in Y$,then we can write
$Y = X_1 \cup \{2n\},\ for some\ X_1 \in D(L^2_{n-1}, i-1)$,that is $Y \in Y_1$.

If 2n $\notin Y$ and 2n-1 $\in Y$,then we can write
$Y = X_2 \cup \{2n-1\},\ for some X_2 \in D(L^2_{n-1} - \{2n-2\}, i-1)$
that is $Y \in Y_2$.

Now suppose that 2n-2 $\in Y$,2n $\notin Y$ and 2n-1 $\notin Y$,then we can write
$Y = X_3 \cup \{2n-3\},\ for some\ X_3 \in D(L^2_{(n-2)}, i-1)$,that is Y $\in Y_3$.

Now suppose that
2n-5 $\in Y$,2n-3 $\notin Y$ and 2n-4 $\notin Y$,we can write
$Y = X_4 \cup \{2n-3\},\ for some\ X_4 \in D(L^2_{n-2} - \{2n-4\}, i-1)$
,
suppose that 2n-3 $\in Y$,2n, 2n-1,
2n-2 $\notin Y$ then we can write
$Y = X_4 \cup \{2n-3\},\ for some\ X_4 \in D(L^2_{n-2} - \{2n-4\}, i-1)$
.
If 2n-4 $\in Y$ 2n, 2n-1,2n-2,2n-3, $\notin Y$ and then 2n-5 $\in Y$,
then we can write
$Y = X_5 \cup \{2n-4\},\ for some\ X_5 \in D(L^2_{(n-3)}, i-1)$

$$D(L^2_n - \{2n\}, i) \subseteq Y_1 \cup Y_2 \cup Y_3 \cup Y_4 \cup Y_5$$
(3.6)

Therefore, we have proved

$$D(L^2_n - \{2n\}, i) = \{X_1 \cup \{2n\} / X_1 \in D(L^2_{n-1}, i-1)\} \cup$$
$$\{X_2 \cup \{2n-1\} / X_2 \in D(L^2_{(n-1)} - \{2n-2\}, i-1)\} \cup$$
$$\{X_3 \cup \{2n-2\} / X_3 \in D(L^2_{n-2}, i-1)\} \cup$$
$$\{X_4 \cup \{2n-3\} / X_4 \in D(L^2_{(n-2)} - \{2n-4\}, i-1)\} \cup$$
$$\{X_5 \cup \{2n-4\} / X_5 \in D(L^2_{n-3}, i-1)\}$$

4 Domination polynomial of a square Ladder (L^2_n).

Let $D(L^2_n, X) = \sum_{i=\lfloor \frac{n}{4} \rfloor + 1}^{n} d(L^2_n, i) x^i$ be the domination polynomial of a Ladder L^2_n .In this section,we study this polynomial.

Theorem 4.1

i) If $D(L^2_n, i)$ is the family of dominating sets with cardinality i of L^2_n ,then

$$d(L^2_n, i) = d(L^2_n - \{2n\}, i-1) + d(L^2_{n-1}, i-1) +$$
$$d(L^2_{(n-1)} - \{2n-2\}, i-1) + d(L^2_{n-2}, i-1) +$$
$$d(L^2_{(n-2)} - \{2n-4\}, i-1) + d(L^2_{n-3}, i-1) + d(L^2_{n-4}, i-1)$$

ii) For every n $\geq$ 7,

$$D(L^2_n, x) = x[\ D(L^2_n - \{2n\}, x) +$$
$$D(L^2_{n-1}, x) + D(L^2_{(n-1)} - \{2n-2\}, x) - \{2n-2\}, x)$$
$$D(L^2_{n-2}, x) + D(L^2_{(n-2)} - \{2n-4\}, x) + D(L^2_{n-3}, x) + D(L^2_{n-4}, x)$$

with the initial values

$D(L^2_1, x) = x^2 + 2x$, $D(L^2_2, x) = x^4 + 4x^3 + 6x^2 + 4x$,
$D(L^2_{3-\{6\}}, x) = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x$,
$D(L^2_3, x) = x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 2x$,
$D(L^2_4 - \{8\}, x) = x^7 + 7x^6 + 21x^5 + 35x^4 + 30x^3 + 18x^2 + 2x$,
$D(L^2_4, x) = x^8 + 8x^7 + 28x^6 + 56x^5 + 70x^4 + 53x^3 + 18x^2$,
$D(L^2_5 - \{10\}, x) = x^9 + 9x^8 + 36x^7 + 84x^6 + 126x^5 + 121x^4 + 67x^3 + 21x^2$

Proof:

i) It follows from theorem 2.7

$$d(L^2_n, i) = d(L^2_n - \{2n\}, i-1) + d(L^2_{n-1}, i-1) + d(L^2_{(n-1)} - \{2n-2\}, i-1) +$$
$$d(L^2_{n-2}, i-1) + d(L^2_{(n-2)} - \{2n-4\}, i-1) + d(L^2_{n-3}, i-1) + d(L^2_{n-4}, i-1)$$

$$\sum d(L^2_n, i)x^i = \sum d(L^2_n - \{2n\}, i-1)x^i + \sum d(L^2_{n-1}, i-1)x^i +$$
$$\sum d(L^2_{(n-1)} - \{2n-2\}, i-1)x^i + \sum d(L^2_{n-2}, i-1)x^i +$$
$$\sum d(L^2_{(n-2)} - \{2n-4\}, i-1)x^i + \sum d(L^2_{n-3}, i-1)x^i + \sum d(L^2_{n-4}, i-1)x^i$$

$$\sum d(L^2_n, i)x^i = x\sum d(L^2_n - \{2n\}, i-1)x^{i-1} + x\sum d(L^2_{n-1}, i-1)x^{i-1} +$$
$$x\sum d(L^2_{(n-1)} - \{2n-2\}, i-1)x^{i-1} + x\sum d(L^2_{n-2}, i-1)x^{i-1} +$$
$$x\sum d(L^2_{(n-2)} - \{2n-4\}, i-1)x^{i-1} + x\sum d(L^2_{n-3}, i-1)x^{i-1} + x\sum d(L^2_{n-4}, i-1)x^{i-1}$$

$$\sum d(L^2_n, i)x^i = x\Big[\sum d(L^2_n - \{2n\}, i-1)x^{i-1} + \sum d(L^2_{n-1}, i-1)x^{i-1} +$$
$$\sum d(L^2_{(n-1)} - \{2n-2\}, i-1)x^{i-1} + \sum d(L^2_{n-2}, i-1)x^{i-1} +$$
$$\sum d(L^2_{(n-2)} - \{2n-4\}, i-1)x^{i-1} + \sum d(L^2_{n-3}, i-1)x^{i-1} +$$
$$\sum d(L^2_{n-4}, i-1)x^{i-1}\Big]$$

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$D(L_n^2, x)$ =

$x[\ D(L_n^2 - \{ 2n \}, x) + D(L_{n-1}^2, x) + D(L_{(n-1)}^2 - \{ 2n-2 \}, x)$

$D(L_{n-2}^2, x) + D(L_{(n-2)}^2 - \{ 2n-4 \}, x) + D(L_{n-3}^2, x) + D(L_{n-4}^2, x)$

]

Theorem 4.2

i) If $S_n = \sum_{i=\left\lfloor \frac{n}{4} \right\rfloor+1}^{n} d(L_n^2, i)$,

$T_n = \sum_{i=\left\lfloor \frac{n}{4} \right\rfloor+1}^{n-1} d(L_n^2 - \{2n\}, i)$ and $R_n = S_n + T_n$

then for every $n \geq 5$,

$R_n = T_{n-1} + S_{n-1} + T_{n-2} + S_{n-2} + T_{n-3} + S_{n-3} + S_{n-4}$

with initial values

T_4 =465, S_4 =234 T_3 =119, S_3 =59, T_2 =31 , S_2 =15 and S_1 =3.

Proof

i) By theorem 3.1, we have

$$S_n = \sum_{i=\left\lfloor \frac{n}{4} \right\rfloor+1}^{n} d(L_n^2, i)$$

$$S_n = \sum_{i=\left\lfloor \frac{n}{4} \right\rfloor+1}^{n} [\ d(L_{n-1}^2, i) + d(L_{n-2}^2, i) + d(L_{n-3}^2, i) + d(L_{n-4}^2, i)\ ]$$

$$S_n = \sum_{i=\left\lfloor \frac{n}{4} \right\rfloor}^{n-1} [\ d(L_{n-1}^2, i-1) + d(L_{n-2}^2, i-1) + + d(L_{n-3}^2, i-1) + d(L_{n-4}^2, i-1)\ ]$$

$$S_n = \sum_{i=\left\lfloor \frac{n}{4} \right\rfloor}^{n-1} d(L_{n-1}^2, i-1) + \sum_{i=\left\lfloor \frac{n}{4} \right\rfloor}^{n-1} d(L_{n-2}^2, i-1) + \sum_{i=\left\lfloor \frac{n}{4} \right\rfloor}^{n-1} d(L_{n-3}^2, i-1) + \sum_{i=\left\lfloor \frac{n}{4} \right\rfloor}^{n-1} d(L_{n-4}^2, i-1)$$

$$S_n = S_{n-1} + S_{n-2} + S_{n-3} + S_{n-4}$$

Also,

$$T_n = \sum_{i=\left\lfloor \frac{n}{4} \right\rfloor+1}^{n-1} d(L_n^2 - \{2n\}, i)$$

$$T_n = \sum_{i=\left\lfloor \frac{n}{4} \right\rfloor+1}^{n-1} [\ d(L_n^2 - \{2n\}, i) + d(L_{n-1}^2 - \{2n-2\}, i) + d(L_{n-2}^2 - \{2n-4\}, i)\ ]$$

$$T_n = \sum_{i=\left\lfloor \frac{n}{4} \right\rfloor}^{n-2} [\ d(L_n^2 - \{2n\}, i) + d(L_{n-1}^2 - \{2n-2\}, i) + d(L_{n-2}^2 - \{2n-4\}, i)\ ]$$

$$T_n = \sum_{i=\left\lfloor \frac{n}{4} \right\rfloor}^{n-2} d(L_n^2 - \{2n\}, i) + \sum_{i=\left\lfloor \frac{n}{4} \right\rfloor}^{n-2} d(L_{n-1}^2 - \{2n-2\}, i) + \sum_{i=\left\lfloor \frac{n}{4} \right\rfloor}^{n-2} d(L_{n-2}^2 - \{2n-4\}, i)$$

$T_n = T_{n-1} + T_{n-2} + T_{n-3}$.

Hence

$$R_n = T_{n-1} + S_{n-1} + T_{n-2} + S_{n-2} + T_{n-3} + S_{n-3} + S_{n-4}$$

Theorem 4.3

The following properties hold for the coefficients of $D(L_n^2, X)$ **and** $D(L_{n-\{2n\}}^2, X)$

i) $d(L_n^2, 2n) = 1$

ii) $d(L_n^2 - \{2n\}, 2n-1) = 1$

iii) $d(L_n^2, 2n-1) = 2n$

iv) $d(L_n^2 - \{2n\}, 2n-2) = 2n-1$

v) $d(L_n^2, 2n-2) = n(2n-1)$

vi) $d(L_n^2 - \{2n\}, 2n-3) = (n-1)(2n-1)$

vii) $d(L_n^2, 2n-3) = \dfrac{2n(n-1)(2n-1)}{3}$

viii) $d(L_n^2 - \{2n\}, 2n-4) = \dfrac{(n-1)(2n-1)(2n-3)}{3}$

Proof:

i) By induction on n.

The result is true for n=1

Therefore (i) and (ii) holds.

ii) Similar to (i)

iii) By induction on n.

The result is true for n=2.

Now suppose that the result is true for all numbers less than ‘ n' and we prove it for n.

By theorem (3.1)

$$d(L_n^2, 2n-1) = d(L_n^2 - \{2n\}, 2n-2) + d(L_{n-1}^2, 2n-2) + d(L_{(n-1)}^2 - \{2n-2\}, 2n-2)$$
$$+ d(L_{n-2}^2, 2n-2) + d(L_{(n-2)}^2 - \{2n-4\}, 2n-2) +$$
$$d(L_{n-3}^2, 2n-2) + d(L_{n-4}^2, 2n-2)$$

=2n—1+1+0+0=2n.

iv) By induction on n.

The result is true for n=2.

Now suppose that the result is true for all numbers less than ‘ n' and we prove it for n.

By theorem (3.1)

$$d(L_n^2 - \{2n\}, 2n-2) = d(L_{n-1}^2, 2n-3) + d(L_{(n-1)}^2 - \{2n-2\}, 2n-3)$$
$$+ d(L_{n-2}^2, 2n-3) + d(L_{(n-2)}^2 - \{2n-4\}, 2n-3) +$$
$$d(L_{n-3}^2, 2n-3) + d(L_{n-4}^2, 2n-3) + d(L_{(n-4)}^2 - \{2n-8\}, 2n-3)$$

=2(n-1)+1+0+0=2n-1.

v) By induction on n.

The result is true for n=2.

Now suppose that the result is true for all numbers less than ‘ n' and we prove it for n.

By theorem (3.1)

$$d(L_n^2, 2n-2) = d(L_n^2 - \{2n\}, 2n-3) + d(L_{n-1}^2, 2n-3) + d(L_{(n-1)}^2 - \{2n-2\}, 2n-3)$$
$$+ \ d(L_{n-2}^2, 2n-3) + d(L_{(n-2)}^2 - \{2n-4\}, 2n-3) +$$
$$d(L_{n-3}^2, 2n-3) + d(L_{n-4}^2, 2n-3)$$

= (n-1)(2n-1)+2n-2+1+0+0

= $2n^2 - 3n + 1 + 2n - 1 = 2n^2 - n = n(2n-1)$.

Hence the proof.

vi) By induction on n.

The result is true for n=2.

Now suppose that the result is true for all numbers less than ‘ n' and we prove it for n.

By theorem (3.1)

$d(L_n^2 - \{2n\}, 2n-3) =$

$d(L_{n-1}^2, 2n-4) + d(L_{(n-1)}^2 - \{2n-2\}, 2n-4) +$

$d(L_{n-2}^2, 2n-4) + d(L_{(n-2)}^2 - \{2n-4\}, 2n-4) +$

$d(L_{n-3}^2, 2n-4) + d(L_{n-4}^2, 2n-4) + d(L_{(n-4)}^2 - \{2n-8\}, 2n-4)$

$= (n-1)(2(n-1)-1) + 2(n-1) + 1 + 0$

$= (n-1)(2n-3) + 2(n-1) + 1$

$= (n-1)(2n-3+2) + 1 = (n-1)(2n-1) + 1$

$= 2n^2 - 3n + 1 = (n-1)(2n-1)$.

Hence the proof.

vii) By induction on n.

The result is true for n=2.

Now suppose that the result is true for all numbers less than ' n' and we prove it for n.

By theorem (3.1)

$d(L_n^2 - \{2n\}, 2n-4) =$

$d(L_{n-1}^2, 2n-5) + d(L_{(n-1)}^2 - \{2n-2\}, 2n-5) +$

$d(L_{n-3}^2, 2n-5) + d(L_{n-4}^2, 2n-5) + d(L_{(n-4)}^2 - \{2n-8\}, 2n-5)$

$= \frac{(n-1)(2(n-1)-1)(2(n-1)-2)}{3} + (n-2)(2(n-1)-1) +$

$2(n-2) + 1$

$= \frac{(n-1)(2n-3)(2n-4)}{3} + (n-2)(2n-3) + 2(n-2) + 1$

$= \frac{1}{3}\left[(n-1)(2n-3)(2n-4) + 3(n-2)(2n-3) + 6(n-2) + 3\right]$

$=$

$\frac{1}{3}\left[(n-1)(2n-3)(2n-4) + 3(n-2)(2n-3) + 3\left[2(n-2)+1\right]\right]$

$= \frac{1}{3}\left[(n-1)(2n-3)(2n-4) + 3(n-2)(2n-3) + 3\left[2(n-2)+1\right]\right]$

$= \frac{1}{3}\left[(n-1)(2n-3)(2n-4) + 3(2n-3)\left[n-2+1\right]\right]$

$= \frac{1}{3}\left[(n-1)(2n-3)(2n-4) + 3(2n-3)(n-1)\right]$

$= \frac{1}{3}\left[(n-1)(2n-3)\left[2n-4+3\right]\right]$

$= \frac{(n-1)(2n-1)(2n-3)}{3}$

Hence the proof.

viii) By induction on n.

The result is true for n=2.

Now suppose that the result is true for all numbers less than ' n' and we prove it for n.

By theorem (3.1)

$d(L_n^2, 2n-3) = d(L_n^2 - \{2n\}, 2n-4) + d(L_{n-1}^2, 2n-4) + d(L_{(n-1)}^2 - \{2n-2\}, 2n-4)$

$+ \quad d(L_{n-2}^2, 2n-4) + d(L_{(n-2)}^2 - \{2n-4\}, 2n-4) +$

$d(L_{n-3}^2, 2n-4) + d(L_{n-4}^2, 2n-4)$

$= \frac{(n-1)(2n-1)(2n-3)}{3} + (n-1)(2(n-1)-1) + \quad (2(n-1)-1) + 1$

$= \frac{1}{3}\left[(n-1)(2n-1)(2n-3) + 3(n-1)(2n-3) + 3(2n-3) + 3\right]$

$= \frac{1}{3}\left[(n-1)(2n-1)(2n-3) + 3(n-1)(2n-3) + 3\left[2n-3+1\right]\right]$

$= \frac{1}{3}\left[(n-1)(2n-1)(2n-3) + 3(n-1)(2n-3) + 3(2n-2)\right]$

$= \frac{1}{3}\left[(n-1)(2n-1)(2n-3) + 3(n-1)(2n-3) + 6(n-1)\right]$

$= \frac{1}{3}\left[(n-1)(2n-1)(2n-3) + 3(n-1)\left[2n-3+2\right]\right]$

$= \frac{1}{3}\left[(n-1)(2n-1)(2n-3) + 3(n-1)(2n-1)\right]$

$= \frac{1}{3}\left[(n-1)(2n-1)\left[2n-3+3\right]\right]$

$= \frac{1}{3}\left[(n-1)(2n-1)2n\right]$

$= \frac{n(2n-1)(2n-2)}{3}$

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
n																	
L_1^2	2	1															
$L_2^2-\{4\}$	3	3	1														
L_2^2	4	6	4	1													
$L_3^2-\{6\}$	5	10	10	5	1												
L_3^2	2	15	20	15	6	1											
$L_4^2-\{8\}$	2	18	35	35	21	7	1										
L_4^2	0	18	53	70	56	28	8	1									
$L_5^2-\{10\}$	0	21	67	121	126	84	36	9	1								
L_5^2	0	15	89	189	247	210	120	45	10	1							
$L_6^2-\{12\}$	0	16	96	269	431	256	330	165	55	11	1						
L_6^2	0	8	109	364	700	887	586	495	220	66	12	1					
$L_7^2-\{14\}$	0	8	103	444	1033	1568	1666	1080	715	286	78	13	1				
L_7^2	0	2	101	537	1472	2600	3234	2946	1795	1001	364	91	14	1			
$L_8^2-\{16\}$	0	2	85	586	1908	3961	5756	6146	4931	2795	1265	455	115	15	1		
L_8^2	0	0	69	636	2459	5848	9711	11903	11080	7877	3960	1520	570	130	16	1	
$L_9^2-\{18\}$	0	0	56	623	2877	7945	15159	21312	22826	18854	12026	5479	1880	700	146	17	1

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